

Self-acceleration and matter content in bicosmology from Noether Symmetries

Mariam Bouhmadi-López^{1,2,3,4}, Salvatore Capozziello^{5,6,7}, and Prado Martín-Moruno⁸

¹*Departamento de Física, Universidade da Beira Interior, 6200 Covilhã, Portugal*

²*Centro de Matemática e Aplicações da Universidade da Beira Interior (CMA-UBI), 6200 Covilhã, Portugal*

³*Department of Theoretical Physics, University of the Basque Country UPV/EHU, P.O. Box 644, 48080 Bilbao, Spain*

⁴*IKERBASQUE, Basque Foundation for Science, 48011 Bilbao, Spain*

⁵*Dipartimento di Fisica, Università di Napoli "Federico II, Compl. Univ. di Monte Sant'Angelo, Via Cinthia 9, I-80126 Napoli, Italy*

⁶*Istituto Nazionale di Fisica Nucleare (INFN) Sez. di Napoli, Compl. Univ. di Monte Sant'Angelo, Via Cinthia 9, I-80126 Napoli, Italy*

⁷*Gran Sasso Science Institute (INFN), Viale F. Crispi 7, I-67100 L'Aquila, Italy*

⁸*Departamento de Física Teórica I, Ciudad Universitaria, Universidad Complutense de Madrid, E-28040 Madrid, Spain*

E-mail: mbl@ubi.pt On leave of absence from UPV and IKERBASQUE, capozziello@na.infn.it, pradomm@ucm.es

ABSTRACT: We consider the existence of Noether symmetries in bigravity cosmologies in order to constrain the material content minimally coupled to the gravitational sector that we are not inhabiting. Interestingly, a Noether symmetry not only constrain the matter content of the universe we do not inhabit but also comes as a sort of bonus on the form of a very interesting dynamics of the universe we live in. In fact, by assuming that our universe is filled with standard matter and radiation, we show that the existence of a Noether symmetry implies the existence of a vacuum energy in our universe that can explain, in a natural way, the current acceleration of the universe. This vacuum energy is intrinsic to the model and can be realized either for a theory that is not properly a bigravity model or for a genuinely bimetric scenario. In fact, it would correspond to a “mono”-universe with a Λ CDM matter symmetry or to a bimetric world where our universe would have once again a Λ CDM matter symmetry while the non-inhabited universe could have a gravitational coupling with a different sign to that in ours. The main point in these pictures is that the Noether symmetry plays a major role into dynamics. The physical consequences are also briefly discussed.

Contents

1	Introduction	1
2	Bic cosmology	3
2.1	Bimetric cosmology	3
2.2	The point-like Lagrangian	6
3	The Noether Symmetry Approach	7
3.1	Non-degenerate point-like Lagrangian	7
3.2	Noether symmetries for bigravity cosmology	9
3.3	Solutions compatible with matter in our universe	10
3.3.1	General Relativity with a cosmological constant	10
3.3.2	Putative bigravity solution	11
3.3.3	Bigravity solution I	12
3.3.4	Bigravity solution II	15
4	The anti-gravitational universe	16
4.1	Cosmological evolution of the first bigravity model	17
4.2	Cosmological evolution of the second bigravity model	18
5	Summary and conclusions	18
A	Symmetric gauge fixing	19

1 Introduction

Bigravity theories, which are models of two mutually interacting dynamical metrics, were initially introduced by Isham, Salam, and Strathdee in the seventies [1]. These theories can be interpreted as describing two universes interacting in a classical way through their gravitational effects [2]. Recently, they have attracted considerable attention since Hassan and Rosen formulated a bigravity theory [3] that is free of the Boulware–Deser ghost [4]. The formulation of this ghost-free bigravity theory was possible thanks to the development of a theory of massive gravity by de Rham, Gabadadze and Tolley which also was potentially stable [5, 6] (see references [7, 8] for a reviews). This massive gravity theory has 5 propagating modes, however, some solutions may still present stability issues related to the scalar polarization [9–12]. In addition, there could be nontrivial gravitational effects in vacuum for massive gravity theories with a Friedmann-Lemaître-Robertson-Walker (FLRW) background metric [13]. Nevertheless, these potential shortcomings would not necessarily affect the ghost-free bigravity theory [13, 14], which could also be considered

to be conceptually favoured against massive gravity since it is a background independent theory.

Cosmological scenarios of the ghost-free bigravity theory were studied in the first place assuming that one of the universes is empty [15–17]. Although these models were able to describe accelerated solutions in absence of a cosmological constant [18, 19], the possible presence of two sets of material content, minimally coupled to each universe, has been also considered [20, 21], as well as perturbations in bimetric cosmologies for both cases [22–28]. It must be noted that assuming the absence of matter in one universe could be a restriction stronger than not to consider its possible existence, as it was emphasized in reference [20].

On the other hand, it must be noted that if the material content of the universe that we are not inhabiting in bigravity were completely arbitrary, then one could re-construct any desired cosmological evolution for our universe by simply fine-tuning that material content which is not directly observable [20]. Thus, it would be desirable to find an argument which would favor a particular kind of material content hidden in the other gravitational sector. With these considerations in mind, we can explore for some general approach fixing the role and the dynamics of matter in bigravity. Specifically, in the context of Extended Theories of Gravity [29, 30] the Noether Symmetry Approach [31] has been proven to be a useful method for obtaining exact solutions in cosmological scenarios [32, 33]. The method consists in assuming the existence of a Noether symmetry which allows to integrate the equations of motion. Because the symmetry does not always exist, the method allows also to select the form of the effective Lagrangians fixing, in particular, the form of couplings and potentials (for more information about this approach and the consequences of the symmetries see [33]). In bigravity, the situation is different, that is: the Lagrangian is already fixed to belong to the family of ghost-free Lagrangians, which only includes four free parameters, and the possible presence of hidden matter in the other gravitational sector entails a great freedom [20]. According to this consideration, the presence of a Noether Symmetry can allow to fix the mater content and the couplings between the two metrics. In this paper, we will consider that the material content should be such that there is a Noether symmetry for the resulting model [33]. Thus, we will apply the Noether Symmetry Approach to bimetric cosmology selecting not only the material content in the hidden gravitational sector (when assuming the presence of a suitable amount of ordinary matter in our universe) but selecting also some of the free parameters appearing in the interaction Lagrangian.

Ghost-free multigravity theories have also been considered [34–36], as well as more general couplings of the matter sector [37–40], general $f(R)$ kinetic terms for the metrics [41], and Lanczos–Lovelock terms [43] in higher dimensional generalizations [42]. Nevertheless, in this paper, we consider a theory of the form

$$S = S_{EH}(g) + \kappa \bar{S}_{EH}(f) + m^2 S_{int}(gf^{-1}) + S_m(g, \phi_\alpha, \nabla_\alpha \phi_\alpha) + \epsilon \bar{S}_m(f, \bar{\phi}_\alpha, \bar{\nabla}_\alpha \bar{\phi}_\alpha), \quad (1.1)$$

where we assume that the interaction term is independent of the derivatives of the metrics (and it is of the particular form introduced in [3]), that $\phi_\alpha \neq \bar{\phi}_\alpha$, $\kappa \neq 0$, $m^2 \neq 0$ and ϵ are different constants. Although the interaction term leading to a ghost-free bimetric gravity theory [3] is the same as the additional terms giving mass to the graviton in the

ghost-free massive gravity theory [5, 6, 44], the characteristics and underlying philosophy of those theories is completely different. As it has been pointed out in [45], if one insists in considering massive gravity as a particular limit of bimetric gravity, that limit would corresponds to setting $\kappa = \epsilon = 0$. Nevertheless, such a limit must be considered with caution as there is not complete continuity in the parameter space of the theory [45].

The paper can be summarized as follows: In section 2, we give the basic formalism of cosmological solutions in bigravity. In section 3, after briefly introducing the Noether Symmetry Approach, we consider what kind of material content should be present in the other gravitational sector in order to have a Noether symmetry in the biuniverse, once the material content of our universe is fixed. In section 4, we discuss the particular atypical solutions that we obtain. We summarize the results in section 5 and relegate some considerations about the definition of the point-like Lagrangian to appendix A.

2 Bicosmology

As bigravity can be interpreted to describe two different universes, it is of special interest to study the cosmological consequences for such a classical bi-universe. In this section, we summarize and rewrite some known results regarding cosmological scenarios of the ghost-free bimetric theory of gravity [3]. In the first place, in the subsection 2.1, we derive the equations of motion from the action of the theory and obtain the modified Friedmann equations by substituting the cosmological ansatz in those equations. In subsection 2.2, we obtain a point-like Lagrangian by considering the cosmological ansatz in the general action of the theory. As we will show, the modified Friedmann equations can also be obtained by varying the point-like Lagrangian, therefore, being both procedures compatible.

2.1 Bimetric cosmology

The interaction Lagrangian of the ghost-free bigravity theory formulated by Hassan and Rosen is a function of γ , implicitly defined as

$$\gamma^\mu{}_\rho \gamma^\rho{}_\nu = g^{\mu\rho} f_{\mu\rho}, \quad (2.1)$$

and where the action reads [3]

$$\begin{aligned} S = & \frac{M_P^2}{2} \int d^4x \sqrt{-g} R(g) + \frac{\kappa M_P^2}{2} \int d^4x \sqrt{-f} R(f) - m^2 \int d^4x \sqrt{-g} \sum_{n=0}^4 \beta_n e_n(\gamma) \\ & + \int d^4x \sqrt{-g} L_m(\phi_\alpha, \nabla_\beta \phi_\alpha) + \int d^4x \sqrt{-f} \bar{L}_m(\bar{\phi}_\alpha, \bar{\nabla}_\beta \bar{\phi}_\alpha). \end{aligned} \quad (2.2)$$

On the above expression, the elementary symmetric polynomials are [46]

$$e_0(\gamma) = 1, \quad (2.3)$$

$$e_1(\gamma) = \text{tr}[\gamma], \quad (2.4)$$

$$e_2(\gamma) = \frac{1}{2} (\text{tr}[\gamma]^2 - \text{tr}[\gamma^2]), \quad (2.5)$$

$$e_3(\gamma) = \frac{1}{6} (\text{tr}[\gamma]^3 - 3 \text{tr}[\gamma] \text{tr}[\gamma^2] + 2 \text{tr}[\gamma^3]), \quad (2.6)$$

$$e_4(\gamma) = \det(\gamma), \quad (2.7)$$

and the fields ϕ_α and $\bar{\phi}_\alpha$ are minimally coupled to g and f , respectively. In addition, M_P is the Planck mass, κ a dimensionless constant while m and β_n are also (free) constants of the model with mass and inverse mass squared dimensions, respectively. It must be noted that although we have not explicitly written a cosmological constant for each kinetic term in action (2.2), those cosmological constants have been absorbed in the e_0 and e_4 terms of the interaction Lagrangian, since they are equivalent to a cosmological constant for the g -space and f -space, respectively [47]. The variation of (2.2) with respect to the two metrics leads to two sets of modified Einstein equations. These are [15, 45]

$$G^\mu{}_\nu = \frac{1}{M_P^2} \left(T^{(m)\mu}{}_\nu + T^\mu{}_\nu \right), \quad (2.8)$$

and

$$\bar{G}^\mu{}_\nu = \frac{1}{\kappa M_P^2} \left(\bar{T}^{(m)\mu}{}_\nu + \bar{T}^\mu{}_\nu \right), \quad (2.9)$$

where

$$\begin{aligned} T^\mu{}_\nu = & -m^2 [\beta_0 + \beta_1 e_1(\gamma) + \beta_2 e_2(\gamma) + \beta_3 e_3(\gamma)] \delta^\mu{}_\nu + m^2 [\beta_1 + \beta_2 e_1(\gamma) + \beta_3 e_2(\gamma)] \gamma^\mu{}_\nu \\ & - m^2 [\beta_2 + \beta_3 e_1(\gamma)] \{\gamma^2\}^\mu{}_\nu + \beta_3 \{\gamma^3\}^\mu{}_\nu, \end{aligned} \quad (2.10)$$

and

$$\begin{aligned} \bar{T}^\mu{}_\nu = & -m^2 \beta_4 - m^2 \sqrt{gf^{-1}} \{[\beta_1 + \beta_2 e_1(\gamma) + \beta_3 e_2(\gamma)] \gamma^\mu{}_\nu \\ & - [\beta_2 + \beta_3 e_1(\gamma)] \{\gamma^2\}^\mu{}_\nu + \beta_3 \{\gamma^3\}^\mu{}_\nu\}. \end{aligned} \quad (2.11)$$

The indexes of equations (2.8) and (2.9) must be raised and lowered using g and f , respectively.

Now, let us consider a cosmological scenario. We assume that the metrics can be written as follows:

$$ds_g^2 = -N(t)^2 dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]. \quad (2.12)$$

and

$$ds_f^2 = -\bar{N}(t)^2 dt^2 + b(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (2.13)$$

where $k = 0, \pm 1$ is the spatial curvature parameter. More general cosmological ansatzs have been also considered in the literature in references [48] and [49]. The modified Friedman equations for each space can be obtained substituting the ansatzs (2.12) and (2.13) in equations (2.8) and (2.9). That can be done either by brute force [15] or noting that (2.12) and (2.13) are related through the generalized Gordon ansatz [50], therefore, one has

$$e_1 = 3\frac{b}{a} + \frac{\bar{N}}{N}, \quad e_2 = 3\frac{b^2}{a^2} + 3\frac{\bar{N}b}{Na}, \quad e_3 = \frac{b^3}{a^3} + 3\frac{\bar{N}b^2}{Na^2}. \quad (2.14)$$

Following any of both procedures, one can obtain the modified Friedmann equations. These are

$$H_g^2 + \frac{k}{a^2} = \frac{1}{3M_P^2} [\rho_{bg}(b/a) + \rho_m(a)], \quad (2.15)$$

and

$$H_f^2 + \frac{k}{b^2} = \frac{1}{3\kappa M_P^2} [\bar{\rho}_{\text{bg}}(a/b) + \bar{\rho}_{\text{m}}(b)], \quad (2.16)$$

where the effective energy density due to the interaction in each space has been defined as [50]

$$\rho_{\text{bg}} = m^2 \left(\beta_0 + 3\beta_1 \frac{b}{a} + 3\beta_2 \frac{b^2}{a^2} + \beta_3 \frac{b^3}{a^3} \right), \quad (2.17)$$

$$\bar{\rho}_{\text{bg}} = m^2 \left(\beta_4 + 3\beta_3 \frac{a}{b} + 3\beta_2 \frac{a^2}{b^2} + \beta_1 \frac{a^3}{b^3} \right), \quad (2.18)$$

and the Hubble parameters are $H_g = \dot{a}/(N a)$ and $H_f = \dot{b}/(\bar{N} b)$, respectively, with $\dot{} \equiv d/dt$.

On the other hand, due to the diffeomorphism invariance of the matter actions appearing in action (2.2), the stress-energy tensors of the matter components of both spaces are conserved. Therefore one has $\dot{\rho}_{\text{m}}/N + 3H_g[1 + w_{\text{m}}(a)]\rho_{\text{m}} = 0$ and $\dot{\bar{\rho}}_{\text{m}}/\bar{N} + 3H_f[1 + \bar{w}_{\text{m}}(b)]\bar{\rho}_{\text{m}} = 0$. Moreover, taking into account the Bianchi identities together with these conservations, the effective stress energy tensors (2.10) and (2.11) must be also conserved. This leads to the Bianchi-inspired constraint [15, 16, 50]

$$\frac{\dot{b}(t)}{\bar{N}(t)} = \frac{\dot{a}(t)}{N(t)}. \quad (2.19)$$

As is already well known this implies that the Hubble parameter of the f -space can be expressed as $H_f = \dot{a}/(N b)$. Therefore, the Friedmann equations of both spaces, equations (2.15) and (2.16), are coupled [15, 16]. This allows us to write the Friedmann equation of the second universe (2.16) as

$$\frac{a^2}{b^2} \left(H_g^2 + \frac{k}{a^2} \right) = \frac{1}{3\kappa M_P^2} [\bar{\rho}_{\text{bg}}(a/b) + \bar{\rho}_{\text{m}}(b)], \quad (2.20)$$

which can be combined with equation (2.15) to remove the term H_g^2 . Considering a general material content in both universes, this is [20]

$$c_4 \Gamma^4 + c_3 \Gamma^3 - \bar{D} \bar{\rho}_{\text{m}}(b) \Gamma^3 + c_2 \Gamma^2 + D \rho_{\text{m}}(a) \Gamma + c_1 \Gamma - c_0 = 0, \quad (2.21)$$

where $\Gamma = b/a$, $c_4 = \beta_3/3$, $c_3 = \beta_2 - \beta_4/(3\kappa)$, $c_2 = \beta_1 - \beta_3/\kappa$, $c_1 = \beta_0/3 - \beta_2/\kappa$, $c_0 = \beta_1/(3\kappa)$, $D = 1/(3m^2)$, $\bar{D} = 1/(3m^2\kappa)$. This equation can be seen as an algebraic equation in b and, therefore, it can be solved to obtain $b(a)$ at least in principle. Solutions for the case in which the second universe is empty are generically easier to obtain, as one can define the quantity $\Gamma = b/a$ as in reference [15] and then equation (2.21) is a quartic equation in Γ (it could even be simpler for models in which some parameters β_i vanish [15]). Once one obtains $b(a)$ from equation (2.21), it can be substituted in the Friedmann equation to study the dynamics of the universe by integrating $a(t)$.

2.2 The point-like Lagrangian

In the previous subsection we have written the general equations for the dynamics of the metric, that is the modified Einstein equations. Then we have restricted our attention to solutions described by two FLRW geometries. Nevertheless, one could have also studied the problem considering a point-like Lagrangian, which leads to dynamics in a minisuperspace. To obtain such a point-like Lagrangian, one has to substitute the cosmological metrics given by equations (2.12) and (2.13) in action (2.2). Therefore, replacing expressions (2.14) in the interaction term of the Lagrangian (2.2) and integrating by parts the Einstein-Hilbert Lagrangian of action (2.2), one gets the following point-like Lagrangian:

$$\begin{aligned} \mathcal{L} = & Na^3 \left[-3M_P^2 \frac{\dot{a}^2}{N^2 a^2} + 3M_P^2 \frac{k}{a^2} - \rho_{\text{bg}}(b/a) - \rho_{\text{m}}(a) \right] \\ & + \bar{N} b^3 \left[-3\kappa M_P^2 \frac{\dot{b}^2}{\bar{N}^2 b^2} + 3\kappa M_P^2 \frac{k}{\bar{b}^2} - \bar{\rho}_{\text{bg}}(a/b) - \bar{\rho}_{\text{m}}(b) \right], \end{aligned} \quad (2.22)$$

with $\rho_{\text{bg}}(b/a)$ and $\bar{\rho}_{\text{bg}}(a/b)$ have been defined in equations (2.17) and (2.18), respectively. This is the same Lagrangian as that presented in reference [14], but we have split the interaction term in $\rho_{\text{bg}}(b/a)$ and $\bar{\rho}_{\text{bg}}(a/b)$ for convenience. It must be noted that the Lagrangian (2.22) is defined in the tangent space, $\mathcal{TQ} \equiv \{N, \bar{N}, a, \dot{a}, b, \dot{b}\}$, coming from the configuration space $\mathcal{Q} \equiv \{N, \bar{N}, a, b\}$.

Varying the Lagrangian (2.22) with respect to N and \bar{N} , we obtain the modified Friedmann equations which are equations (2.15) and (2.16), respectively. Now, the variation with respect to the scale factor a leads to

$$-2 \frac{1}{a} \frac{d^2 a}{dt_{ca}^2} - \left[\frac{1}{a} \frac{da}{dt_{ca}} \right]^2 - \frac{k}{a^2} = -\frac{1}{M_P^2} \left(\rho_{\text{bg}} + \frac{a}{3} \rho'_{\text{bg}} + \frac{1}{3} \frac{b^3 \bar{N}}{a^2} \bar{\rho}'_{\text{bg}} + \rho_{\text{m}} + \frac{a}{3} \rho'_{\text{m}} \right), \quad (2.23)$$

with $dt_{ca} \equiv N dt$ and $' \equiv \partial/\partial a$. The conservation equation can be written as $\rho_{\text{m}} + \frac{a}{3} \rho'_{\text{m}} = -p_{\text{m}}$. Therefore, if the interaction term vanishes (for $m^2 = 0$), the acceleration equation (2.23) will be equivalent to consider the derivative of Friedmann equation and the conservation of the fluid, as in usual General Relativity (GR); i.e. the standard Raychaudhuri equation. By similarity to what happens in GR, we can define an effective pressure, p_{bg} , as follows

$$p_{\text{bg}} = - \left(\rho_{\text{bg}} + \frac{a}{3} \rho'_{\text{bg}} + \frac{1}{3} \frac{b^3 \bar{N}}{a^2} \bar{\rho}'_{\text{bg}} \right). \quad (2.24)$$

Combining the derivative of the Friedmann equation (2.15) with equation (2.23), one obtains

$$\frac{N}{\bar{N}} = \frac{\dot{a}}{\dot{b}}, \quad (2.25)$$

which is exactly the Bianchi-inspired constraint (2.19). Therefore, this procedure is equivalent to that presented in the previous subsection. Nevertheless, it can be noted that the Lagrangian (2.22) is degenerate in the tangent space $\mathcal{TQ} \equiv \{N, \bar{N}, a, \dot{a}, b, \dot{b}\}$. This fact can be problematic when applying certain procedures, as the Noether Symmetry Approach or the quantization of the corresponding Hamiltonian. Thus, one can interpret the fact

that the Bianchi inspired constraint (2.19) should be obtained by deriving the equations of motion and requiring their compatibility, as being a consequence of the large number of equations of motion provided by an initial degenerate Lagrangian. As we will see in the next section, one can define a non-degenerate Lagrangian containing the information encoded in equation (2.19) from the beginning.

3 The Noether Symmetry Approach

As it is well known, in order to apply the Noether Symmetry Approach, one needs a non-degenerate point-like Lagrangian $\mathcal{L}(q_i, \dot{q}_i)$ which is independent of time. Then, one supposes the existence of a Noether symmetry which implies that [31]

$$L_X \mathcal{L}(q_i, \dot{q}_i) = 0, \quad (3.1)$$

where X is a vector field defined on the tangent space $\mathcal{TQ} \equiv \{q^i, \dot{q}^i\}$, that is

$$X = \xi^i(q^i) \frac{\partial}{\partial q^i} + \dot{\xi}^i(q^i) \frac{\partial}{\partial \dot{q}^i}, \quad (3.2)$$

and L_X is the Lie derivative along the direction X . Equation (3.1) gives rise to a system of partial differential equations whose solution is not unique. The solutions of equation (3.1) fix the vector components $\xi^i(q^i)$ (and consequently $\dot{\xi}^i(q^i)$) and the functional form of the Lagrangian $\mathcal{L}(q_i, \dot{q}_i)$, that is couplings and potentials (see [31] for details). Besides, the Lagrangian, and then the dynamics, are associated to conserved quantities Σ_0 that can be used to integrate the equations of motion, being

$$\frac{d}{dt} \left(\xi^i \frac{\partial \mathcal{L}}{\partial \dot{q}^i} \right) = L_X \mathcal{L}, \quad (3.3)$$

and then

$$\Sigma_0 = \xi^i \frac{\partial \mathcal{L}}{\partial \dot{q}^i}, \quad (3.4)$$

as a consequence of $L_X \mathcal{L} = 0$. Furthermore, the energy function

$$E_{\mathcal{L}}(q_i, \dot{q}_i) = \frac{\partial \mathcal{L}}{\partial \dot{q}^i} \dot{q}^i - \mathcal{L}, \quad (3.5)$$

is conserved as well. The right hand side of equation (3.5) corresponds to the Hamiltonian of the system, \mathcal{H} , and given that $\partial \mathcal{H} / \partial t = -\partial \mathcal{L} / \partial t$, it follows immediately that $E_{\mathcal{L}}$ is conserved, whenever \mathcal{L} does not depend on time. Therefore, the use of this equation further simplifies the problem, and analytic solutions can easily be found. This approach has been used several times in cosmology giving exact solutions of physical interest (see for example [51–54]).

3.1 Non-degenerate point-like Lagrangian

As the Lagrangian (2.22) is independent of \dot{N} and \ddot{N} , the determinant of the Hessian of the Lagrangian vanishes and the Lagrangian is degenerate¹ [31]. The easiest way to find

¹The Hessian of the Lagrangian is defined as $H_{ij} = \partial^2 \mathcal{L} / (\partial \dot{q}^i \partial \dot{q}^j)$.

a non-degenerate point-like Lagrangian, therefore, consists in removing the dependence on N and \overline{N} from \mathcal{L} . In the first place, we note that, without loss of generality, we can fix $N = 1$, such that the cosmic time of the g -universe will be t . Because in this theory we have only one global invariance under changes of coordinates, this choice already fixes the temporal gauge freedom and, therefore, we cannot freely choose \overline{N} . Nevertheless, it can be noted that \overline{N} must be such that the Lagrangian contains the information from the Bianchi-inspired constraint (2.19) from the beginning, that is,

$$N = 1, \quad \overline{N} = \frac{\dot{b}}{\dot{a}}. \quad (3.6)$$

Therefore, one has

$$\begin{aligned} \mathcal{L} = & - \left[3M_P^2 a \dot{a}^2 - 3M_P^2 k a + a^3 \rho_{\text{bg}}(b/a) + a^3 \rho_{\text{m}}(a) \right] \\ & - \left[3\kappa M_P^2 b \dot{a} \dot{b} - 3\kappa M_P^2 k b \frac{\dot{b}}{\dot{a}} + b^3 \frac{\dot{b}}{\dot{a}} \overline{\rho}_{\text{bg}}(a/b) + b^3 \frac{\dot{b}}{\dot{a}} \overline{\rho}_{\text{m}}(b) \right]. \end{aligned} \quad (3.7)$$

It has to be emphasized that this Lagrangian is not symmetric under inter-change of the g -universe and f -universe, as it has been the case until fixing the gauge freedom. The reason is that the gauge fixing (3.6) breaks the symmetry between both gravitational sectors, and now one universe can be distinguished from the other. In particular, we are choosing to express the physics in terms of our cosmic time. That is not incidental and it will have consequences in the system of differential equations produced by the condition (3.1). As it is shown in the appendix A, this symmetry breaking could be necessary to obtain a non-degenerate and nontrivial point-like Lagrangian, as it is suggested by the simplest symmetric gauge fixing introduced and discussed in the appendix A.

In order to check that the Lagrangian (3.7) describes the same physical situation as (2.22), one should obtain the same information as in the previous section. In fact, varying the Lagrangian with respect to b and after some simplifications, one obtains

$$\left(\frac{\dot{a}}{\dot{b}} \right)^2 + \frac{k}{b^2} = \frac{1}{3\kappa M_P^2} [\overline{\rho}_{\text{bg}}(a/b) + \overline{\rho}_{\text{m}}(b)]. \quad (3.8)$$

This is just the Friedmann equation of the second universe, equation (2.16), with H_f written in terms of \dot{a} through equation (3.6). Moreover, varying with respect to a , simplifying, substituting equation (3.8) when needed, and simplifying again, one gets

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = -\frac{1}{M_P^2} [p_{\text{m}}(a) + p_{\text{bg}}(b/a)], \quad (3.9)$$

with p_{bg} given by equation (2.24), which is equation (2.23). Thus, the system of equations is equivalent to that obtained from the Lagrangian (2.22).

On the other hand, as the non degenerate point-like Lagrangian has no explicit dependence on t , the energy (3.5) is conserved, that is

$$E_{\mathcal{L}} = -3 M_P^2 a^3 \left[\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - \frac{\rho_{\text{bg}} + \rho_{\text{m}}}{3 M_P^2} \right] - 3\kappa M_P^2 b^3 \frac{\dot{b}}{\dot{a}} \left[\frac{\dot{a}^2}{b^2} + \frac{k}{b^2} - \frac{\overline{\rho}_{\text{bg}} + \overline{\rho}_{\text{m}}}{3\kappa M_P^2} \right] = 0, \quad (3.10)$$

where we know that this function vanishes for compatibility with the Friedmann equation. Although equation (3.10) could suggest that the energy of each universe is conserved separately, this is not the case because writing $E_{\mathcal{L}} = E_{\mathcal{L}_1} + E_{\mathcal{L}_2}$, $E_{\mathcal{L}_1}$ cannot be obtained using only \mathcal{L}_1 (it has contributions from \mathcal{L}_2), and vice versa. Anyway, it is important to emphasize that the consideration of the equation implied by $E_{\mathcal{L}} = 0$ is equivalent to one of the equations of motion of the system, and it would be easier to calculate in general.

3.2 Noether symmetries for bigravity cosmology

Now that we have a non-degenerate, time independent, point-like Lagrangian, we can search for Noether symmetries. A general vector field on $\mathcal{TQ} \equiv \{a, \dot{a}, b, \dot{b}\}$ can be expressed as

$$X = \xi(a, b) \frac{\partial}{\partial a} + \eta(a, b) \frac{\partial}{\partial b} + \frac{d\xi(a, b)}{dt} \frac{\partial}{\partial \dot{a}} + \frac{d\eta(a, b)}{dt} \frac{\partial}{\partial \dot{b}}, \quad (3.11)$$

and a Noether symmetry exists if condition (3.1) is satisfied. Explicitly the vector (3.11) is to be applied to the point-like Lagrangian (3.7) giving the condition (3.1). The output is a system of partial differential equations for $\xi(a, b)$ and $\eta(a, b)$, obtained by imposing to be zero the coefficients of the terms \dot{a}^2 , \dot{b}^2 , $\dot{a}\dot{b}$, \dot{b}/\dot{a} , \dot{b}^2/\dot{a}^2 and \dot{b}^0/\dot{a}^0 (see reference [31] for details). These are:

$$\xi + 2a\xi' + \kappa b\eta' = 0, \quad (3.12)$$

$$\xi^s = 0, \quad (3.13)$$

$$\eta + \frac{2}{\kappa}a\xi^s + b\xi' + b\eta^s = 0, \quad (3.14)$$

$$\eta B^s + (\eta^s - \xi')B + \overline{C}'\xi + \overline{C}^s\eta + \overline{C}(\eta^s - \xi') + 3\kappa M_P^2 k(-\eta + b\xi' - b\eta^s) = 0, \quad (3.15)$$

$$(-3\kappa M_P^2 b k + \overline{C} + B)\xi^s = 0, \quad (3.16)$$

and

$$A'\xi + B\eta' + C^s\eta + C'\xi + \overline{C}\eta' - 3M_P^2 k\xi - 3\kappa M_P^2 k b\eta' = 0, \quad (3.17)$$

respectively, where we have defined $' \equiv \partial/\partial a$, $^s \equiv \partial/\partial b$, $C(a, b) = a^3 \rho_{\text{bg}}(b/a)$, $\overline{C}(a, b) = b^3 \overline{\rho}_{\text{bg}}(a/b)$, $A(a) = a^3 \rho_{\text{m}}(a)$ and $B(b) = b^3 \overline{\rho}_{\text{m}}(b)$. It is worth noticing that, in the Lagrangian (3.7), we have not introduced any particular form for the functions $\rho_{\text{m}}(a)$ and $\overline{\rho}_{\text{m}}(b)$. Thus, we can wonder whether there are some functions $\rho_{\text{m}}(a)$ and $\overline{\rho}_{\text{m}}(b)$ for which the system is compatible; in other words, we can consider whether there is a material content in each universe which is compatible with the existence of a Noether symmetry. In summary, the unknown functions² of the system of equations (3.12)-(3.17) are $\xi(a, b)$, $\eta(a, b)$, $\rho_{\text{m}}(a)$ and $\overline{\rho}_{\text{m}}(b)$; therefore, the system has more equations than unknown functions and can be incompatible. If there were no solution, then one would conclude that there is no material content which allows the existence of a Noether symmetry in bigravity (at least, there would not be a Noether symmetry assuming Einstein-Hilbert gravitational terms and the fulfillment of the equivalence principle).

²It is worth noticing that if the configuration space \mathcal{Q} has dimension n , the number of differential equations emerging from $L_X \mathcal{L} = 0$ is $1 + n(n+1)/2$. In our case, we obtain six equations instead of four, because we have two more unknown functions that are $\rho_{\text{m}}(a)$ and $\overline{\rho}_{\text{m}}(b)$.

Equation (3.13) simply implies

$$\xi^s = 0 \Rightarrow \xi = \xi(a). \quad (3.18)$$

This condition makes (3.16) trivially satisfied. Taking into account that ξ is independent of b in equation (3.12), one has

$$\eta'(a, b) = \frac{\eta'_0(a)}{b} \Rightarrow \eta(a, b) = \frac{\eta_0(a)}{b} + c(b), \quad (3.19)$$

being $\eta_0(a)$ a function of a which should be determined using the other equations. Introducing equations (3.19) and (3.18) in equation (3.14), one gets

$$\frac{c(b)}{b} + c^s(b) = -\xi'(a) \equiv -\xi_0, \quad (3.20)$$

which must, therefore, be equal to a constant, we denoted as $-\xi_0$. Then, this implies

$$\xi(a) = \xi_0 a + \phi, \quad \eta(a, b) = \frac{\eta_0(a)}{b} - \frac{\xi_0}{2} b + \frac{\delta}{b}, \quad (3.21)$$

where ϕ and δ are arbitrary constants. Now, considering again equation (3.12), we have

$$\xi(a) = \xi_0 a + \phi, \quad \eta(a, b) = -\frac{3}{2\kappa} \xi_0 \frac{a^2}{b} - \frac{\phi}{\kappa} \frac{a}{b} + \frac{\delta}{b} - \frac{\xi_0}{2} b. \quad (3.22)$$

Therefore, the most general $\xi(a)$ and $\eta(a, b)$ compatible with equations (3.12), (3.13), (3.14) and (3.16) are given by (3.22). In order to obtain these functions, we have not imposed any restrictions on $A(a)$ and $B(b)$, but we have still two equations of the system. Thus, the system would be compatible if there are functions $A(a)$ and $B(b)$ which are solutions of equations (3.15) and (3.17) with $\xi(a)$ and $\eta(a, b)$ given by equation (3.22).

3.3 Solutions compatible with matter in our universe

Let us now take into account that in our universe there is only matter (dust) and radiation, that is

$$A(a) = A_0 + \frac{A_1}{a}. \quad (3.23)$$

Therefore, we want to find which material content, $B(b)$, should be present in the other universe to have a Noether symmetry for the bi-universal dynamics, given by equations (3.11) and (3.22). Thus, we look for functions $B(b)$ which are solutions of both equations (3.15) and (3.17), once expressions (3.22) and (3.23) have been introduced in these equations.

3.3.1 General Relativity with a cosmological constant

The first solution is given by

$$\xi_0 = 0, \quad A_1 = 0, \quad \beta_0 = \beta_1 = \beta_2 = \beta_3 = 0. \quad (3.24)$$

Thus, the only non-vanishing parameter in the interaction Lagrangian is β_4 , implying that interaction between both sectors reduces to a cosmological constant in the second

universe. In this case, the dynamics of both universes is decoupled and have the same spatial geometry, i.e. the same value of k (which is arbitrary). The second universe is empty, since the cosmological constant appearing in the geometric term cancel the contribution coming from $B(b)$. That is

$$B(b) = -b^3 m^2 \beta_4 \Rightarrow \bar{\rho}_m(b) = -m^2 \beta_4, \quad (3.25)$$

$$\rho_{\text{bg}}(a/b) = 0, \quad \bar{\rho}_{\text{bg}}(b/a) = m^2 \beta_4. \quad (3.26)$$

Moreover, one can substitute the parameters in equations (3.24) and (3.25) into equation (2.21) to study the solution of this system. This leads to

$$\rho_m(a) \Gamma = 0. \quad (3.27)$$

As $\rho_m = A_0/a^3$, this implies that $\Gamma = 0$ and, therefore, $b = 0$. Assuming that this condition is preserved in time, one has $\dot{b} = 0$ which implies $\bar{N} = 0$ and, therefore, there is no second gravitational sector. In this case, not only the interaction is equivalent to a cosmological constant, but also there is only one gravitational sector filled with dust matter. Thus, there is no bigravity solutions for this model.

3.3.2 Putative bigravity solution

The second solution for the system (3.15) and (3.17) is given by

$$\xi_0 = 0, \quad A_1 = 0, \quad \beta_1 = \beta_3 = 0, \quad \beta_0 = \frac{3}{\kappa} \beta_2, \quad \delta = 0. \quad (3.28)$$

In this case, the interaction between both gravitational sectors is not equivalent to a cosmological constant, since $\beta_2 \neq 0$. Indeed one has a non-trivial interaction affecting the dynamics of our universe, which is given by

$$\rho_{\text{bg}} = 3\beta_2 m^2 \left(\frac{1}{\kappa} + \frac{b^2}{a^2} \right). \quad (3.29)$$

That is, a cosmological constant component plus a purely bimetric term. It must be noted that the solution requires $A_1 = 0$, that is, no radiation in the universe. Our universe contains, of course, relativistic matter; nevertheless, one could consider that the state in which a pure Noether symmetry is approached, the universe is in its asymptotic past or future; i.e. during a “pre-inflationary” phase to be followed by a standard inflationary era or at the very late-time dark energy era. In both situations, the radiation contribution can be negligible as compared with the material content. Moreover, this solution is a solution valid for any k .

In the other universe, the material content compatible with the existence of a Noether symmetry is

$$\bar{\rho}_m = m^2 (3\kappa\beta_2 - \beta_4). \quad (3.30)$$

Thus, the other universe only contains vacuum energy³. As the material content of the second universe is equivalent to the consideration of a cosmological constant, the solution for this particular model corresponds to the “minimal model” discussed in [15] plus an explicitly cosmological constant contribution. This model was called minimal in reference [15] because, as $\beta_1 = \beta_3 = 0$, the only nonlinear interaction term in both universes is the quadratic term (it should not be confused with the minimal model introduced in reference [19]). In this case one has that some coefficients appearing in equation (2.21) are fixed, $c_0 = c_2 = c_4 = 0$. Nevertheless, in the particular model we are considering, β_0 and β_2 are not independent, thus one additionally has $c_1 = 0$. Therefore, even if in other alternative theories of gravity the Noether Symmetry Approach was used not only to single out a particular model but also to find out exact solutions [31–33], in this case it is particularly simple to obtain the analytic solution using equation (2.21) once we know which model we want to study due to the potential presence of a Noether symmetry. Therefore, substituting these values of the parameters and equation (3.30) in equation (2.21), it can be seen that the solutions for this model are given by

$$\rho_m \frac{b}{a} = 0. \quad (3.31)$$

As in the previous case, since $\rho_m = A_0/a^3$, we have $b = 0$. Thus, assuming again $\dot{b} = 0$, one can conclude that this model is not compatible with a second gravitational sector. The effect of requiring the existence of a Noether symmetry leads to a general relativistic world with a cosmological constant term, fixed by β_2 , and, therefore, suitable to describe, for example, the current acceleration of our universe.

3.3.3 Bigravity solution I

The last set of solutions of the system (3.15) and (3.17) is

$$\phi = 0, \quad A_1 = 0, \quad \beta_1 = \beta_3 = 0, \quad \beta_0 = \frac{6}{\kappa}\beta_2, \quad k = 0, \quad \delta = 0, \quad (3.32)$$

which leads to a non-trivial interaction given by

$$\rho_{\text{bg}}(b/a) = 3\beta_2 m^2 \left(\frac{2}{\kappa} + \frac{b^2}{a^2} \right). \quad (3.33)$$

The content of the other universe is expressed as

$$\bar{\rho}_m = -m^2 \beta_4. \quad (3.34)$$

³One could be surprised about this result since, given the symmetry of the interaction term, one would naively expect the same symmetry regarding the material content of the universes. Nevertheless, in order to obtain a non-degenerate Lagrangian we have broken the original symmetry of the Lagrangian, since otherwise no dynamical evolution could have been obtained as it is explained in the appendix A. This can be interpreted as suggesting that the symmetry of the problem is broken when one considers that we live in one of the universes and, as a consequence, the Noether symmetry would be associated with a cosmological model in which the couple of universes contain different kind of material content (one is empty and the other has ordinary matter).

As in the previous case, given the simplicity of the model assigned by the parameters (3.32) and (3.34), we can study the dynamics by solving equation (2.21) to check the consistency of the model before searching for a Noether symmetry. In this case, substituting (3.32) and (3.34) in equation (2.21), we obtain

$$\Gamma^2 = -\frac{\rho_m(a)}{3m^2\beta_2} - \frac{1}{\kappa}. \quad (3.35)$$

Hence, in order to have a real function b , one needs to consider $\beta_2 < 0$ and/or $\kappa < 0$. On the other hand, considering equation (3.35) in the Friedmann equation (2.15), or equivalently in equation (2.20), with the definitions (2.17) and (2.18), one obtains

$$H_g^2 = \frac{m^2\beta_2}{\kappa M_P^2}, \quad (3.36)$$

that implies that κ and β_2 should have the same sign, therefore, both parameters should be negative. From equations (3.35) and (3.36) it can be seen that, in this case, both a and b can be well defined. Some comments regarding the physics of the other universe are included in section 4.

Let us study this case in more detail. Defining $M_{P_2}^2 = -\kappa M_P^2 > 0$ and integrating equation (3.36), we obtain

$$a(t) = a_0 \exp\left(m\sqrt{|\beta_2|/M_{P_2}^2}t\right), \quad (3.37)$$

Thus, even if we have considered only the presence of standard matter in our universe ($A_0 \neq 0$), the evolution (3.37) is exactly de Sitter. The scale factor (3.37) can be rewritten as

$$a(t) = a_0 \exp\left(\sqrt{\Lambda_{\text{eff}}/3}t\right), \quad \Lambda_{\text{eff}} = 3m^2 \frac{|\beta_2|}{M_{P_2}^2}. \quad (3.38)$$

Therefore, the effective cosmological constant which appears in the dynamic evolution, equation (3.38), is different of the cosmological constant induced by the interaction term, (2.17), which is $\Lambda_g = m^2\beta_0$. Indeed, one has

$$\Lambda_{\text{eff}} = \frac{\Lambda_g}{2M_P^2}, \quad (3.39)$$

where in the case that one considers an explicit cosmological constant Λ , the term Λ_g has to be replaced by $\Lambda_g - \Lambda$. It has to be emphasized that we are considering our universe approaching the state in which there is a Noether symmetry with time evolution. That is, it tends to a state where the effect of matter in the dynamics is negligible even before Ω_m is small enough to assume that the material content is diluted enough to be negligible for the evolution.

On the other hand, the components of the Noether vector field (3.11) assume the explicit forms

$$\xi(a) = \xi_0 a, \quad \eta(a, b) = \frac{3M_P^2}{2M_{P_2}^2} \xi_0 \frac{a^2}{b} - \frac{\xi_0}{2} b. \quad (3.40)$$

Following the considerations in reference [31], the Noether vector field can be written in a simplified form as

$$\tilde{X} = \frac{\partial}{\partial v}, \quad (3.41)$$

by defining a new par of variables $\{u, v\}$ such that v is cyclic; i.e. the Lagrangian \mathcal{L} can be recast in a form where only kinetic terms in \dot{v} appear. This change of variables is always possible as soon as a Noether symmetry is present and the condition (3.4) is satisfied. The change of variables can be realized assuming a regular transformation $u = u(a, b)$ and $v = v(a, b)$ where the Jacobian of transformation is different from zero. Explicitly, we have

$$\xi(a) \frac{\partial u}{\partial a} + \eta(a, b) \frac{\partial u}{\partial b} = 0, \quad \xi(a) \frac{\partial v}{\partial a} + \eta(a, b) \frac{\partial v}{\partial b} = 1 \quad (3.42)$$

and obtain

$$u(a, b) = u_0 a \left(b^2 - \frac{M_P^2}{M_{P_2}^2} a^2 \right), \quad v(a, b) = \frac{1}{\xi_0} \ln(a) + v_0 a \left(b^2 - \frac{M_P^2}{M_{P_2}^2} a^2 \right), \quad (3.43)$$

where u_0 and v_0 are integration constants. Reverting (3.43) and assuming, without lost of generality, $v_0 = u_0 = \xi_0 = 1$, we obtain

$$a = \exp(v - u), \quad b = \sqrt{u \exp(u - v) + \frac{M_P^2}{M_{P_2}^2} \exp[2(v - u)]}. \quad (3.44)$$

The Lagrangian, in terms of these new variables, assume the form $\tilde{\mathcal{L}}(u, \dot{u}, \dot{v})$ where the new variable v is cyclic. Therefore, from equation (3.41), the constant of motion which is given by

$$\Sigma_0 = \frac{\partial \tilde{\mathcal{L}}}{\partial \dot{v}}, \quad (3.45)$$

which is the same as that defined in equation (3.4) but in the new variables. Once obtained, this quantity can be expressed in terms of the scale factor of both universes as

$$\Sigma_0 = 3 M_{P_2}^2 a b \dot{b} - \frac{3}{2} [M_{P_2}^2 b^2 + M_P^2 a^2] + 3\beta_2 m^2 \frac{a^3 b \dot{b}}{\dot{a}^2} - \frac{3\beta_2 m^2}{2} \frac{a^2}{\dot{a}} \left(3 \frac{M_P^2}{M_{P_2}^2} a^2 - b^2 \right) \quad (3.46)$$

The fact that this quantity is conserved can be used to integrate the equation of motions. In our case, however, they were easy to integrate so one could check that Σ_0 is indeed conserved using the solutions. Taking into account equations (3.37) and (3.44), equation (3.46) simplifies to

$$\Sigma_0 = -\frac{A_0 M_{P_2}}{m |\beta_2|^{1/2}} = -\frac{A_0}{m} \sqrt{3/\Lambda_{\text{eff}}}. \quad (3.47)$$

In summary, the presence of the conserved quantity fixes the value of the cosmological constant Λ_{eff} , and gives rise to a de Sitter evolution.

3.3.4 Bigravity solution II

The last set of solutions of the system (3.15) and (3.17) is given by

$$\phi = 0, \quad A_1 = 0, \quad \beta_1 = \beta_3 = 0, \quad \beta_0 = \frac{\beta_2}{\kappa}, \quad k = \frac{2\beta_2 m^2 \delta}{M_P^2 \xi_0}. \quad (3.48)$$

For this case, the interaction term in our universe has the same form as the one given in equation (3.33), that is

$$\rho_{\text{bg}}(b/a) = 3\beta_2 m^2 \left(\frac{2}{\kappa} + \frac{b^2}{a^2} \right), \quad (3.49)$$

but now the material content of the other universe is

$$\bar{\rho}_{\text{m}}(b) = -m^2 \beta_4 + \frac{6\kappa m^2 \beta_2 \delta}{\xi_0} b^{-2}, \quad (3.50)$$

which is equivalent to consider a cosmological constant contribution plus a spatial curvature contribution. Therefore, from equation (2.21), we get (considering A_0 positive, see Eq. (3.22))

$$b = \sqrt{\frac{2\delta}{\xi_0} - \frac{A_0}{3m^2\beta_2} \frac{1}{a} - \frac{1}{\kappa} a^2}, \quad (3.51)$$

which can be introduced in the Friedmann equation (2.15) to obtain

$$H_g^2 = \frac{\beta_2 m^2}{\kappa M_P^2}. \quad (3.52)$$

Assuming that the state with a Noether symmetry is approached at late times in our universe, that is for large a , and taking into account equation (3.51) and (3.52) one needs to consider negative values⁴ of β_2 and κ , and one can define again $M_{P_2}^2 = -\kappa M_P^2$. As in the previous case, we obtain

$$a(t) = a_0 \exp\left(\sqrt{\Lambda_{\text{eff}}/3} t\right), \quad \Lambda_{\text{eff}} = 3m^2 \frac{|\beta_2|}{M_{P_2}^2}, \quad (3.53)$$

although in this case this accelerating universe could have a non-vanishing spatial curvature as shown in equation (3.48), its contribution to the dynamics is cancelled by the interaction term.

In this case, the vector field associated with the Noether symmetry, is given by

$$\xi(a) = \xi_0 a, \quad \eta(a, b) = -\frac{\xi_0}{2} b + \frac{3M_P^2 \xi_0}{2M_{P_2}^2} \frac{a^2}{b} + \frac{\delta}{b}. \quad (3.54)$$

⁴In this case one could also have well-defined solutions for positive values of β_2 and κ restricting attention to small values of the scale factor a . We are interested, however, in the situation in which the solution having a Noether symmetry is approached at late times. Notice that a positive value of β would imply a minimum value for the scale factor which would lead to an avoidance of the Big Bang singularity in our universe. Nevertheless, as we said before, we are mainly interested on the late-time behaviour of the universe.

For the the cyclic variable, one can again perform the transformation $\{a, b\} \rightarrow \{u, v\}$ obtaining

$$\begin{aligned} u(a, b) &= u_0 a \left(b^2 - \frac{M_P^2}{M_{P_2}^2} a^2 - \frac{2\delta}{\xi_0} \right), \\ v(a, b) &= \frac{1}{\xi_0} \ln(a) + v_0 a \left(b^2 - \frac{M_P^2}{M_{P_2}^2} a^2 - \frac{2\delta}{\xi_0} \right), \end{aligned} \quad (3.55)$$

where u_0 and v_0 are integration constants. Assuming $v_0 = u_0 = \xi_0 = 1$ in (3.55), one obtains

$$\begin{aligned} a &= \exp(v - u), \\ b &= \sqrt{u \exp(u - v) + \frac{M_P^2}{M_{P_2}^2} \exp[2(v - u)] + 2\delta}. \end{aligned} \quad (3.56)$$

Expressing the point-like Lagrangian in terms of $u(a, b)$ and $v(a, b)$, it can be checked that v is the cyclic variable. As above, there is a conserved quantity which, taking into account solutions (3.51) and (3.53), reduces to

$$\Sigma_0 = -\frac{A_0 M_{P_2}}{m |\beta_2|^{1/2}} = -\frac{A_0}{m} \sqrt{3/\Lambda_{\text{eff}}}, \quad (3.57)$$

as in the previous case.

4 The anti-gravitational universe

Before discussing the physics of the second universe, it should be emphasized that the bi-universal interpretation of solutions of bigravity theory is mainly based on considering this theory as a fundamental representation of reality. If one wants to consider that bigravity theory is nothing more than an effective description of the physics of our universe at a given range of energies (coming from a more fundamental theory), then one could ignore the physics of the other universe, since one would not consider it as a physical realizable world. In this section, however, we will try to study the consequences of assuming the existence of two universes for the physics of the universe that we are not inhabiting.

The solution obtained in the previous section implies that the gravitational coupling for each universe is given by

$$G_g = \frac{1}{8\pi M_P^2}, \quad G_f = -\frac{1}{8\pi M_{P_2}^2}, \quad (4.1)$$

respectively. Thus, whereas gravity is attractive in our universe, it is repulsive in the other universe. When the bi-universe has reached the state with a Noether symmetry studied in the previous section, it would not be any material content in the second universe to feel this anti-gravitational field, since equation (3.34) implies that there is only vacuum energy in the second universe. However, one could consider that it was some material content in

earlier cosmological phases, which has been diluted by the expansion. This matter would follow geodesics of the metric $f_{\mu\nu}$, since the matter fields are minimally coupled to this metric. Nevertheless, the way in which the material content (together with the coupling between both gravitational sectors) would curve the geometry is “the contrary” than in our universe, in the sense that

$$G^\mu{}_\nu = 8\pi G_g \left(T^{(m)\mu}{}_\nu + T^\mu{}_\nu \right), \quad \overline{G}^\mu{}_\nu = -8\pi |G_f| \left(\overline{T}^{(m)\mu}{}_\nu + \overline{T}^\mu{}_\nu \right). \quad (4.2)$$

Therefore, it seems that in the second universe one would have a repulsive gravitational field, but all particles would “fall” the same way in this field in agreement with the equivalence principle. It is worth noticing that the $(-)$ sign in front of $\overline{T}^{(m)\mu}{}_\nu$ can be eliminated by introducing a parameter ϵ on the last term of the action (2.2), where $\epsilon = -1$. In such a way, the symmetry between the two universes is fully restored.

4.1 Cosmological evolution of the first bigravity model

Regarding the cosmic evolution, taking into account equations (3.35) and (3.37), one can obtain the scale factor of this universe. This is

$$b(t) = b_0 \exp(\alpha t) \sqrt{1 + \gamma \exp(-3\alpha t)}, \quad (4.3)$$

where

$$b_0 = a_0 \frac{M_P}{M_{P_2}}, \quad \alpha = \frac{m |\beta_2|^{1/2}}{M_{P_2}}, \quad \gamma = \frac{A_0}{3M_P^2 \alpha^2 a_0^3}. \quad (4.4)$$

This scale factor can be written in terms of the cosmic time τ to be suitably interpreted. This cosmic time of this universe is given by

$$\tau = \int \overline{N} dt = \int \frac{\dot{b}}{\dot{a}} dt. \quad (4.5)$$

Deriving equations (3.37) and (3.38), and then integrating (4.5), this cosmic time is

$$\tau = \tau_0 \left\{ 3\alpha t + \sqrt{1 + \gamma e^{-3\alpha t}} + 2 \ln \left(1 + \sqrt{1 + \gamma e^{-3\alpha t}} \right) + C \right\}, \quad (4.6)$$

where

$$\tau_0 = \frac{b_0 M_{P_2}}{3a_0 m |\beta_2|^{1/2}}, \quad (4.7)$$

and C is an integration constant which can be chosen to impose some origin between both times (as $\tau(0) = 0$). Although a complete analytic expression of $b(\tau)$ is not trivially obtained from equations (4.3) and (4.6), it can be obtained that, for large t , one has

$$b \approx b_0 \exp \left[\sqrt{\frac{\overline{\Lambda}_{\text{eff}}}{3}} (\tau - \tau_*) \right], \quad (4.8)$$

where

$$\overline{\Lambda}_{\text{eff}} = \frac{3m^2 |\beta_2|}{M_P^2}, \quad (4.9)$$

which, taking into account equation (3.38), can be written as

$$\bar{\Lambda}_{\text{eff}} = \frac{M_{P_2}^2}{M_P^2} \Lambda_{\text{eff}}, \quad (4.10)$$

and

$$\tau_* = \frac{1}{\sqrt{3\bar{\Lambda}_{\text{eff}}}} \left[\sqrt{1+\gamma} + 2 \ln \left(1 + \sqrt{1+\gamma} \right) + C \right]. \quad (4.11)$$

Therefore, although the evolution of the second universe is not exactly de Sitter when there is a Noether symmetry, it will approach a de Sitter behaviour asymptotically.

4.2 Cosmological evolution of the second bigravity model

In this case, taking into account equations (3.53) and (3.51), one has

$$b(t) = \sqrt{b_1 \exp(2\alpha t) + b_2 \exp(-\alpha t) + 2\xi_0}, \quad (4.12)$$

with

$$b_1 = a_0^2 \frac{M_P^2}{M_{P_2}^2}, \quad b_2 = \frac{A_0}{3|\beta_2|m^2 a_0}, \quad \alpha = \frac{m|\beta_2|^{1/2}}{M_{P_2}}. \quad (4.13)$$

Although we have not being able to obtain an analytic expression for $\tau(t)$ in this case, it can be noted that, for large t , the first term in (4.12) dominates; therefore, the late time behaviour should be similar to that obtained in the previous section; i.e. the second universe will be again asymptotically de Sitter in the future.

5 Summary and conclusions

Bigravity theories are those theories of gravitation where two metrics, mutually interacting, are considered. They are mainly interesting because represent a natural approach to avoid background dependence and ghosts in massive gravity and in other fundamental theories of gravitational interaction.

Here, we have considered the cosmology emerging from bigravity and the possibility to determine cosmic evolution by searching for Noether symmetries. With respect to the other results present in literature, here we have adopted another strategy. Instead of determining the form of the point-like Lagrangian and then searching for exact solutions by the Noether Symmetry Approach, we have determined the matter content and the evolution of the two interacting universes asking for the existence of the Noether symmetry. In this sense, besides being a method to select exact solutions, the Noether symmetry results in a physical quantity capable of restoring the symmetry between the two universes, to determine their material content, and, finally, to fix the amount of acceleration at early and late times. This last result is related to the fact that an effective cosmological constant is given by the conserved quantity Σ_0 . According to this picture, the existence of Noether symmetries is a selection rule capable of discriminating between physically viable or non-viable cosmological models. The philosophy is the same discussed in reference [33] for the wave function of the universe in quantum cosmology: the presence of Noether symmetries allows to select

observable universes by the Hartle criterion; the absence of Noether symmetries gives rise to non-correlated variables and then to non-observable universes.

In a forthcoming paper, we will consider possible observational signatures of the present approach in bigravity and the quantum cosmology of the present model following, for example, the approach used in reference [55] within a Palatini type of model that can be regarded as a bimetric scenario.

Acknowledgements

The work of MBL is supported by the Portuguese Agency “Fundação para a Ciência e Tecnologia” through an Investigador FCT Research contract, with reference IF/01442/2013/CP1196/CT0001. She also wishes to acknowledge the partial support from the Basque government Grant No. IT592-13 (Spain) and FONDOS FEDER under grant FIS2014-57956-P (Spanish government). This research work is supported by the Portuguese grant UID/MAT/00212/2013. SC acknowledges INFN Sez. di Napoli (Iniziative Specifiche QGSKY and TEONGRAV). PMM acknowledges financial support from the Spanish Ministry of Economy and Competitiveness through the postdoctoral training contract FPD1-2013-16161, and the project FIS2014-52837-P. The authors acknowledge the COST Action CA15117 (CANTATA).

A Symmetric gauge fixing

We could have chosen a gauge fixing in the Lagrangian (2.22) without breaking the symmetry under inter-change of both universe. The most simple gauge fixing in agreement with equation (2.19) is

$$N = \dot{a}, \quad \text{and} \quad \overline{N} = \dot{b}. \quad (\text{A.1})$$

In this case the point-like Lagrangian is

$$\begin{aligned} \mathcal{L} = & - \left[3M_P^2 a \dot{a} (1 - k) + a^3 \dot{a} \rho_{\text{bg}}(b/a) + a^3 \dot{a} \rho_{\text{m}}(a) \right] \\ & - \left[3M_f^2 b \dot{b} (1 - k) + b^3 \dot{b} \overline{\rho}_{\text{bg}}(a/b) + b^3 \dot{b} \overline{\rho}_{\text{m}}(b) \right]. \end{aligned} \quad (\text{A.2})$$

The equations of motion of this Lagrangian can be obtained by considering the variation with respect to a and the variation with respect to b . It can be seen that both cases lead to

$$\overline{\rho}_{\text{bg}}(a/b) = \frac{a^3}{b^3} \rho_{\text{bg}}^s(b/a), \quad (\text{A.3})$$

which is simply an identity satisfied by the effective energy densities given by (2.17) and (2.18). Moreover, as all the terms in the Lagrangian (A.2) are linear either in \dot{a} or in \dot{b} , the energy function (3.5) is exactly zero, and the constraint of constant energy is trivial. Thus, by fixing a symmetric gauge in the point-like Lagrangian, we have lost the information about the dynamics of the system contained in the degenerate Lagrangian (2.22).

References

- [1] C. J. Isham, A. Salam and J. A. Strathdee, “F-dominance of gravity”, *Phys. Rev. D* **3** (1971) 867.
- [2] T. Damour and I. I. Kogan, “Effective Lagrangians and universality classes of nonlinear bigravity”, *Phys. Rev. D* **66** (2002) 104024 [hep-th/0206042].
- [3] S. F. Hassan and R. A. Rosen, “Bimetric Gravity from Ghost-free Massive Gravity”, *JHEP* **1202** (2012) 126 [arXiv:1109.3515 [hep-th]].
- [4] D. G. Boulware, S. Deser, “Can gravitation have a finite range?”, *Phys. Rev. D* **6** (1972) 3368.
- [5] C. de Rham and G. Gabadadze, “Generalization of the Fierz-Pauli Action”, *Phys. Rev. D* **82** (2010) 044020 [arXiv:1007.0443 [hep-th]].
- [6] C. de Rham, G. Gabadadze, and A. J. Tolley, “Resummation of Massive Gravity”, *Phys. Rev. Lett.* **106** (2011) 231101 [arXiv:1011.1232 [hep-th]].
- [7] C. de Rham, “Massive Gravity”, *Living Rev. Rel.* **17** (2014) 7 [arXiv:1401.4173 [hep-th]].
- [8] A. Schmidt-May and M. von Strauss, “Recent developments in bimetric theory”, *J. Phys. A* **49** (2016) no.18, 183001 doi:10.1088/1751-8113/49/18/183001 [arXiv:1512.00021 [hep-th]].
- [9] M. Fasiello and A. J. Tolley, “Cosmological perturbations in Massive Gravity and the Higuchi bound”, *JCAP* **1211** (2012) 035 [arXiv:1206.3852 [hep-th]].
- [10] S. Deser and A. Waldron, “Acausality of Massive Gravity”, *Phys. Rev. Lett.* **110** (2013) 111101 [arXiv:1212.5835 [hep-th]].
- [11] A. H. Chamseddine and V. Mukhanov, “Hidden Ghost in Massive gravity”, *JHEP* **1303** (2013) 092 [arXiv:1302.4367 [hep-th]].
- [12] P. Guarato and R. Durrer, “Perturbations for massive gravity theories”, *Phys. Rev. D* **89** (2014) 084016 [arXiv:1309.2245 [gr-qc]].
- [13] P. Martín-Moruno and M. Visser, “Is there vacuum when there is mass? Vacuum and non-vacuum solutions for massive gravity”, *Class. Quant. Grav.* **30** (2013) 155021 [arXiv:1301.2334 [gr-qc]].
- [14] M. Fasiello and A. J. Tolley, “Cosmological Stability Bound in Massive Gravity and bigravity”, *JCAP* **1312** (2013) 002 [arXiv:1308.1647 [hep-th]].
- [15] M. von Strauss, A. Schmidt-May, J. Enander, E. Mortsell and S. F. Hassan, “Cosmological Solutions in bimetric Gravity and their Observational Tests”, *JCAP* **1203** (2012) 042 [arXiv:1111.1655 [gr-qc]].
- [16] M. S. Volkov, “Cosmological solutions with massive gravitons in the bigravity theory”, *JHEP* **1201** (2012) 035 [arXiv:1110.6153 [hep-th]].
- [17] D. Comelli, M. Crisostomi, F. Nesti, and L. Pilo, “FRW Cosmology in Ghost Free Massive Gravity from bigravity”, *JHEP* **1203** (2012) 067 [arXiv:1111.1983 [hep-th]].
- [18] Y. Akrami, T. S. Koivisto and M. Sandstad, “Accelerated expansion from ghost-free bigravity: a statistical analysis with improved generality”, *JHEP* **1303** (2013) 099 [arXiv:1209.0457 [astro-ph.CO]].
- [19] F. Koennig, A. Patil and L. Amendola, “Viable cosmological solutions in massive bimetric gravity”, *JCAP* **1403** (2014) 029 [arXiv:1312.3208 [astro-ph.CO]].

- [20] S. Capozziello and P. Martín-Moruno, “Bounces, turnarounds and singularities in bimetric gravity,” *Phys. Lett. B* **719** (2013) 14 [arXiv:1211.0214 [gr-qc]].
- [21] K. Aoki and K. i. Maeda, “Cosmology in ghost-free bigravity theory with twin matter fluids: The origin of dark matter”, *Phys. Rev. D* **89** (2014) no.6, 064051 [arXiv:1312.7040 [gr-qc]].
- [22] D. Comelli, M. Crisostomi and L. Pilo, “Perturbations in Massive Gravity Cosmology”, *JHEP* **1206** (2012) 085 [arXiv:1202.1986 [hep-th]].
- [23] D. Comelli, M. Crisostomi and L. Pilo, “FRW Cosmological Perturbations in Massive bigravity”, *Phys. Rev. D* **90** (2014) 084003 [arXiv:1403.5679 [hep-th]].
- [24] A. De Felice, A. E. Gümrükçüoğlu, S. Mukohyama, N. Tanahashi and T. Tanaka, “Viable cosmology in bimetric theory”, *JCAP* **1406** (2014) 037 [arXiv:1404.0008 [hep-th]].
- [25] F. Könnig and L. Amendola, “Instability in a minimal bimetric gravity model”, *Phys. Rev. D* **90** (2014) 044030 [arXiv:1402.1988 [astro-ph.CO]].
- [26] A. R. Solomon, Y. Akrami and T. S. Koivisto, “Linear growth of structure in massive bigravity”, *JCAP* **1410** (2014) 066 [arXiv:1404.4061 [astro-ph.CO]].
- [27] F. Koennig, Y. Akrami, L. Amendola, M. Motta and A. R. Solomon, “Stable and unstable cosmological models in bimetric massive gravity”, *Phys. Rev. D* **90** (2014) 124014 [arXiv:1407.4331 [astro-ph.CO]].
- [28] Y. Akrami, S. F. Hassan, F. Könnig, A. Schmidt-May and A. R. Solomon, “Bimetric gravity is cosmologically viable”, *Phys. Lett. B* **748** (2015) 37 [arXiv:1503.07521 [gr-qc]].
- [29] S. Capozziello and M. De Laurentis, “Extended Theories of Gravity”, *Phys. Rept.* **509** (2011) 167 [arXiv:1108.6266 [gr-qc]].
- [30] S. Nojiri and S. D. Odintsov, “Unified cosmic history in modified gravity: from $F(R)$ theory to Lorentz non-invariant models”, *Phys. Rept.* **505** (2011) 59 [arXiv:1011.0544 [gr-qc]].
- [31] S. Capozziello, R. de Ritis, C. Rubano and P. Scudellaro, “Noether symmetries in cosmology”, *La Rivista del Nuovo Cimento* **4** (1996) 1.
- [32] S. Capozziello, P. Martín-Moruno, and C. Rubano, “Dark energy and dust matter phases from an exact $f(R)$ -cosmology model”, *Phys. Lett. B* **664** (2008) 12 [arXiv:0804.4340 [astro-ph]].
- [33] S. Capozziello, M. De Laurentis and S. D. Odintsov, “Hamiltonian dynamics and Noether symmetries in Extended Gravity Cosmology”, *Eur. Phys. J. C* **72** (2012) 2068, [arXiv:1206.4842 [gr-qc]].
- [34] K. Hinterbichler and R. A. Rosen, “Interacting Spin-2 Fields”, *JHEP* **1207** (2012) 047 [arXiv:1203.5783 [hep-th]].
- [35] M. Lüben, Y. Akrami, L. Amendola and A. R. Solomon, “Cosmology with three interacting spin-2 fields”, *Phys. Rev. D* **94** (2016) no.4, 043530 [arXiv:1604.04285 [astro-ph.CO]].
- [36] N. Tamanini, E. N. Saridakis and T. S. Koivisto, “The Cosmology of Interacting Spin-2 Fields”, *JCAP* **1402** (2014) 015 [arXiv:1307.5984 [hep-th]].
- [37] Y. Akrami, T. S. Koivisto, D. F. Mota and M. Sandstad, “Bimetric gravity doubly coupled to matter: theory and cosmological implications”, *JCAP* **1310** (2013) 046 [arXiv:1306.0004 [hep-th]].
- [38] C. de Rham, L. Heisenberg and R. H. Ribeiro, “On couplings to matter in massive (bi-)gravity”, *Class. Quant. Grav.* **32** (2015) 035022 [arXiv:1408.1678 [hep-th]].

- [39] A. E. Gumrukcuoglu, L. Heisenberg, S. Mukohyama and N. Tanahashi, “Cosmology in bimetric theory with an effective composite coupling to matter”, JCAP **1504** (2015) no.04, 008 [arXiv:1501.02790 [hep-th]].
- [40] L. Heisenberg, “More on effective composite metrics”, Phys. Rev. D **92** (2015) 023525 [arXiv:1505.02966 [hep-th]].
- [41] S. Nojiri and S. D. Odintsov, “Ghost-free $F(R)$ bigravity and accelerating cosmology”, Phys. Lett. B **716** (2012) 377 [arXiv:1207.5106 [hep-th]].
- [42] T. Q. Do, “Higher dimensional massive bigravity”, Phys. Rev. D **94** (2016) no.4, 044022 [arXiv:1604.07568 [gr-qc]].
- [43] S. F. Hassan, A. Schmidt-May and M. von Strauss, “Bimetric theory and partial masslessness with Lanczos–Lovelock terms in arbitrary dimensions”, Class. Quant. Grav. **30** (2013) 184010 [arXiv:1212.4525 [hep-th]].
- [44] S. F. Hassan, R. A. Rosen and A. Schmidt-May, “Ghost-free Massive Gravity with a General Reference Metric”, JHEP **1202** (2012) 026 [arXiv:1109.3230 [hep-th]].
- [45] V. Baccetti, P. Martín-Moruno and M. Visser, “Massive gravity from bimetric gravity”, Class. Quant. Grav. **30** (2013) 015004 [arXiv:1205.2158 [gr-qc]].
- [46] I. G. Macdonald, “Symmetric Functions and Hall Polynomials”, second edition (Oxford: Clarendon Press (1995)).
- [47] V. Baccetti, P. Martín-Moruno and M. Visser, “Null Energy Condition violations in bimetric gravity”, JHEP **1208** (2012) 148 [arXiv:1206.3814 [gr-qc]].
- [48] H. Nersisyan, Y. Akrami and L. Amendola, “Consistent metric combinations in cosmology of massive bigravity”, Phys. Rev. D **92** (2015) no.10, 104034 [arXiv:1502.03988 [gr-qc]].
- [49] C. García-García, A. L. Maroto and P. Martín-Moruno, “Cosmology with moving bimetric fluids”, arXiv:1608.06493 [gr-qc].
- [50] V. Baccetti, P. Martín-Moruno and M. Visser, “Gordon and Kerr-Schild ansatz in massive and bimetric gravity”, JHEP **1208** (2012) 108 [arXiv:1206.4720 [gr-qc]].
- [51] S. Capozziello, S. Nesseris, L. Perivolaropoulos, “Reconstruction of the Scalar-Tensor Lagrangian from a LCDM Background and Noether Symmetry”, JCAP 0712 (2007) 009, [arXiv:0705.3586 [astro-ph]].
- [52] S. Capozziello, M. De Laurentis, R. Myrzakulov, “Noether Symmetry Approach for teleparallel-curvature cosmology”, Int. J. Geom. Meth. Mod. Phys. **12** (2015), 1550095, [arXiv:1412.1471 [gr-qc]].
- [53] S. Basilakos, S. Capozziello, M. De Laurentis, A. Paliathanasis, M. Tsamparlis, “Noether symmetries and analytical solutions in $f(T)$ cosmology: A complete study”, Phys. Rev. D **88** (2013) 103526, [arXiv:1311.2173 [gr-qc]].
- [54] S. Capozziello, A. De Felice, “ $f(R)$ cosmology by Noether’s symmetry”, JCAP 0808 (2008) 016, [arXiv:0804.2163 [gr-qc]].
- [55] M. Bouhmadi-López and C. Y. Chen, “Towards the Quantization of Eddington-inspired-Born-Infeld Theory”, arXiv:1609.00700 [gr-qc], accepted for publication in JCAP.